Week 4 Worksheet Thursday - Implicit Differentiation

Instructions. Discuss with your group mates and do the following problems. You are not expected to finish all the problems. :)

Topic: Derivative Computing & Implicit Differentiation

1. Compute the Derivatives of the following functions (No need to use definition.)

   (a) $(\frac{1}{x^3} + e^{2x})^{100}$

   (b) $y = \sqrt[3]{x^5}e^{2x^2}$

   (c) $y = x^2$. Compute $\left(\frac{dy}{dx}\right)^2$ and $\frac{d^2y}{dx^2}$

$y = (x^{-3} + e^{2x})^{100}$

(a) $y' = 100(x^{-3}+e^{2x})^{99}(-3x^{-4} + e^{2x}\cdot 2)$

(b) $y = x^{\frac{5}{3}}e^{2x^2}$

$y' = \frac{5}{3}x^{\frac{2}{3}}e^{2x^2} + x^{\frac{5}{3}}e^{2x^2}\cdot 4x$

(c) $\frac{dy}{dx} = 2x$ $\quad \left(\frac{dy}{dx}\right)^2 = 4x^2$ $\quad \frac{d^2y}{dx^2} = 2$

2. *Let's do this problem together. :)*
   Find $\frac{dy}{dx}$ given $xy + x^2y + (y+1)^2 = 1$.

$y + x\frac{dy}{dx} + 2xy + x^2\frac{dy}{dx} + 2(y+1)\frac{dy}{dx} = 0$

$(x + x^2 + 2y + 2)\frac{dy}{dx} = -y - 2xy$

$\frac{dy}{dx} = \frac{-y-2xy}{x+x^2+2y+2}$

Now it's time for you to practice!

3. Given $3x^2 - y^3 = 2$. Find value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at point $(1,1)$.

$6x - 3y^2\frac{dy}{dx} = 0$

Plug in $(1,1)$

$\Rightarrow 6 - 3\frac{dy}{dx}\Big|_{(1,1)} = 0$

$\Rightarrow \frac{dy}{dx}\Big|_{(1,1)} = 2$

$3y^2\frac{dy}{dx} = 6x$ ← Take another $\frac{d}{dx}$ to this eq.

$6 - 6y\frac{dy}{dx}\cdot\frac{dy}{dx} - 3y^2\frac{d^2y}{dx^2} = 0$

$6 - 6\cdot 2\cdot 2 - 3\frac{d^2y}{dx^2} = 0$

$\frac{d^2y}{dx^2} = -6$

4. Find the slope of the tangent line at $(2,1)$ to the curve

$$x^2 + xy + 4y^2 = 10$$

$2x + y + x\frac{dy}{dx} + 8y\frac{dy}{dx} = 10$

Plug in $(2,1)$

$4 + 1 + 2\frac{dy}{dx} + 8\frac{dy}{dx} = 10$

$10\frac{dy}{dx} = -5$

$\frac{dy}{dx} = -\frac{1}{2}$

5* *[More Challenging]* Let $y(x)$ be defined implicitly by

$$e^{xy} + (1+x)^3 + (1+y)^3 + y = 3$$

Find the tangent line to the graph of $y(x)$ at a point $x = y = 0$.

$e^{xy}\cdot[y + x\frac{dy}{dx}] + 3(1+x)^2 + 3(1+y)^2\frac{dy}{dx} + \frac{dy}{dx} = 0$

Plug in $x = y = 0$

$3 + 3\frac{dy}{dx} + \frac{dy}{dx} = 0$

$4\frac{dy}{dx} = -3$

$\frac{dy}{dx} = -\frac{3}{4}$.